



## **Cambridge International AS & A Level**

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CENTRE  
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### **FURTHER MATHEMATICS**

**9231/13**

Paper 1 Further Pure Mathematics 1

**October/November 2020**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

### **INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

- 1** The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$ , where  $a$  and  $b$  are positive constants.

(a) The matrix  $\mathbf{M}$  represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

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The unit square in the  $x$ - $y$  plane is transformed by  $\mathbf{M}$  onto parallelogram  $OPQR$ .

(b) Find, in terms of  $a$  and  $b$ , the matrix which transforms parallelogram  $OPQR$  onto the unit square. [2]

It is given that the area of  $OPQR$  is  $2\text{cm}^2$  and that the line  $x+3y=0$  is invariant under the transformation represented by  $\mathbf{M}$ .

- (c) Find the values of  $a$  and  $b$ .

[5]

- 2 (a) Use standard results from the List of Formulae (MF19) to show that

$$\sum_{r=1}^n (7r+1)(7r+8) = an^3 + bn^2 + cn,$$

where  $a$ ,  $b$  and  $c$  are constants to be determined.

[3]

- (b)** Use the method of differences to find  $\sum_{r=1}^n \frac{1}{(7r+1)(7r+8)}$  in terms of  $n$ . [4]

- (c) Deduce the value of  $\sum_{r=1}^{\infty} \frac{1}{(7r+1)(7r+8)}$ . [1]

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- 3 The cubic equation  $x^3 + cx + 1 = 0$ , where  $c$  is a constant, has roots  $\alpha, \beta, \gamma$ .

- (a) Find a cubic equation whose roots are  $\alpha^3$ ,  $\beta^3$ ,  $\gamma^3$ .

[3]

- (b) Show that  $\alpha^6 + \beta^6 + \gamma^6 = 3 - 2c^3$ .

[3]

- (c) Find the real value of  $c$  for which the matrix  $\begin{pmatrix} 1 & \alpha^3 & \beta^3 \\ \alpha^3 & 1 & \gamma^3 \\ \beta^3 & \gamma^3 & 1 \end{pmatrix}$  is singular. [5]

- 4** The points  $A$ ,  $B$ ,  $C$  have position vectors

$$-\mathbf{i} + \mathbf{j} + 2\mathbf{k}, \quad -2\mathbf{i} - \mathbf{j}, \quad 2\mathbf{i} + 2\mathbf{k},$$

respectively, relative to the origin  $O$ .

- (a) Find the equation of the plane  $ABC$ , giving your answer in the form  $ax + by + cz = d$ . [5]

- (b) Find the perpendicular distance from  $O$  to the plane  $ABC$ .

[2]

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- (c) Find the acute angle between the planes  $OAB$  and  $ABC$ .

[4]

10

- 5** Prove by mathematical induction that, for every positive integer  $n$ ,

$$\frac{d^{2n-1}}{dx^{2n-1}}(x \sin x) = (-1)^{n-1} \left( x \cos x + (2n-1) \sin x \right).$$

[7]



- 6 The curve  $C$  has equation  $y = \frac{x^2 + x - 1}{x - 1}$ .

(a) Find the equations of the asymptotes of  $C$ .

[3]

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- (b) Show that there is no point on  $C$  for which  $1 < y < 5$ .

[4]

- (c) Find the coordinates of the intersections of  $C$  with the axes, and sketch  $C$ .

[3]

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- (d) Sketch the curve with equation  $y = \left| \frac{x^2 + x - 1}{x - 1} \right|$ .

[2]

- 7 (a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^{\frac{5}{2}} = 4xy(x^2 - y^2)$$

has polar equation  $r = \sin 4\theta$ .

[4]

The curve  $C$  has polar equation  $r = \sin 4\theta$ , for  $0 \leq \theta \leq \frac{1}{4}\pi$ .

- (b) Sketch C and state the equation of the line of symmetry. [3]

(c) Find the exact value of the area of the region enclosed by  $C$ . [4]

- (d) Using the identity  $\sin 4\theta \equiv 4 \sin \theta \cos^3 \theta - 4 \sin^3 \theta \cos \theta$ , find the maximum distance of  $C$  from the line  $\theta = \frac{1}{2}\pi$ . Give your answer correct to 2 decimal places. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.





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